

If $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0}$
 or
 $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$
 Then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$
 provided that the latter limit exists.

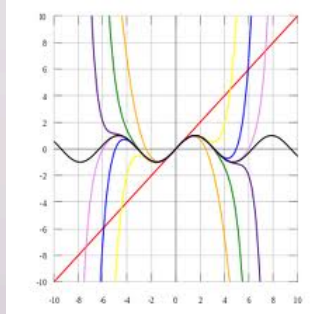
$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$
 $= \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$

$\ln(x) = \int_1^x \frac{1}{t} dt \Rightarrow \ln(2) = \int_1^2 \frac{1}{t} dt \approx 0.69315$

$\int u dv = uv - \int v du$

where it comes from:
 $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$
 put into reverse:
 $\int \frac{d}{dx}(uv) = \int u \frac{dv}{dx} + \int v \frac{du}{dx}$
 and then rearrange:
 $\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$

The Taylor Approximation to a Function

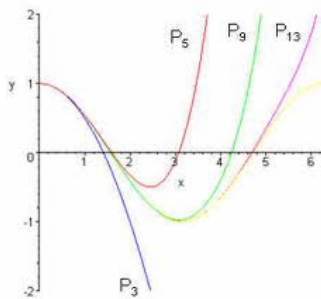


Taylor Approximations to a Function

Many math problems that occur in applications cannot be solved exactly, like $\int_0^b \sin(x^2) dx$. We need to approximate them.

Taylor Polynomial of order n (based at a)

$$P_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$



EX 1 For $f(x) = e^{-3x}$, find the Maclaurin polynomial of order 4 and approximate $f(0.12)$.

Lagrange Error for Taylor Polynomials

We know $f(x) = P_n(x) + R_n(x)$.

$$R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!} (x-a)^{n+1}$$

$c \in (a, x)$ (or $c \in (x, a)$)
if $a < x$ (if $x < a$)

EX 2 Find the error in estimating $f(0.12)$ in the last example, $f(x) = e^{-3x}$.

EX 3 Find a good bound for the maximum value of $\left| \frac{4c}{c+4} \right|$ given $c \in [0,1]$.

EX 4 Find a good bound for the maximum value of $\left| \frac{c^2 - c}{\cos c} \right|$ given $c \in [0, \pi/4]$.

EX 5 Find n such that the Maclaurin polynomial for $f(x) = e^x$ has $f(1)$ approximated to five decimal places, i.e. $|R_n(1)| \leq 0.00005$.