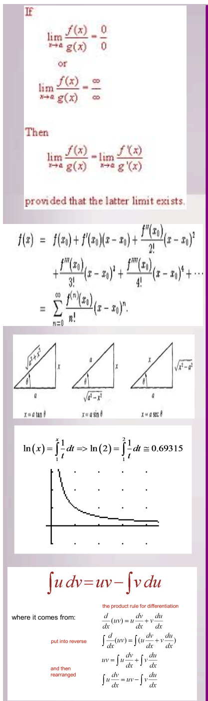
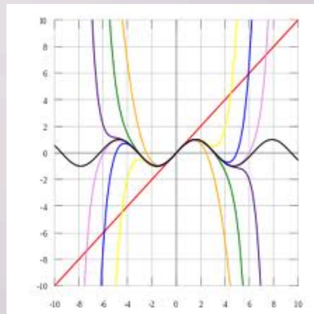


The Taylor Approximation to a Function

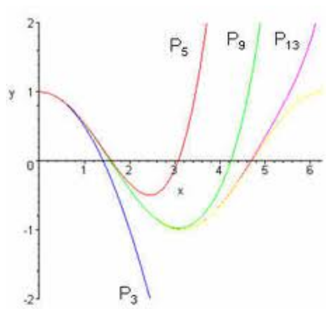


Taylor Approximations to a Function

Many math problems that occur in applications cannot be solved exactly, like $\int_0^b \sin\left(x^2\right)dx$. We need to approximate them.

Taylor Polynomial of order n (based at a)

$$P_n(x)=f(a)+f'(a)\ (x-a)+\frac{f''(a)}{2!}(x-a)^2+...+\frac{f^{(n)}(a)}{n!}(x-a)^n$$



EX 1 For $f(x) = e^{-3x}$, find the Maclaurin polynomial of order 4 and approximate $f(0.12)$.

Lagrange Error for Taylor Polynomials

We know $f(x) = P_n(x) + R_n(x)$.

$$R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$$

$c \in (a, x)$ if $a < x$ (or $c \in (x, a)$ if $x < a$)

EX 2 Find the error in estimating $f(0.12)$ in the last example, $f(x) = e^{-3x}$.

EX 3 Find a good bound for the maximum value of $\left|\frac{4c}{c+4}\right|$ given $c \in [0,1]$.

EX 4 Find a good bound for the maximum value of $\left|\frac{c^2 - c}{\cos c}\right|$ given $c \in [0, \pi / 4]$.

EX 5 Find n such that the Maclaurin polynomial for $f(x) = e^x$ has $f(1)$ approximated to five decimal places, i.e. $|R_n(1)| \leq 0.000005$.